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Mathematical Model for Torsional Vibration Analysis in Internal Combustion Engines

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Abstract—*The scope of this paper is the study of the crankshaft torsional vibration phenomenon in internal combustion engines. The formulation, based on state equation solution with system steady state response calculation performed by transition state matrix and the convolution integral. The analysis considers a rubber and a viscous damper assembled to the crankshaft front-end. From the torsional vibrations analysis, it is possible to obtain the dynamic loading on each crankshaft section and these loads can be applied as boundary conditions in a finite element model to predict the safety factor of the component and compare the system behavior with rubber and viscous damping options. By this way, it is possible to emphasize the importance of the torsional vibrations analysis on the structural dimensioning of the crankshafts.*

Keywords: Torsional Vibrations, Crankshaft, Damper.

I. Literature Review

Johnston, P. R. and Shusto, L. M. [1], developed a technique to predict the behavior of the torsional vibrations in internal combustion engines at transient and steady state regime by the modal superposing method. The results are compared to the measured ones.

Some systems can present excessive vibrations on specific speeds. Draminsky was one of the first researchers who studied these phenomena [2]. Hestermann and Stone [3] concluded that these unexpected large angular displacements in multiples of the engine speed occur due to the variable inertia characteristics of the crank-mechanism.

In the past, the effects of the internal combustion engines variable inertia were considered to be negligible and were disregarded from the calculations. Recently, these secondary effects were verified and checked and they are responsible per many crankshaft structural failures. Paricha, M. S. [4], included these effects on the previous Draminsky´s studies and concluded that, in some cases, the interaction of these secondary forces can be extremely dangerous for the crankshafts.

Brusa et al. [5] studied the effect of the non-constant moments of inertia on torsional vibration calculations. The introduction of functions taking into account the variation of the inertia over the crank throw angular position shall be considered, mainly in cases of large displacement engines, where the masses of the piston and connecting rods are significantly large when compared to the other components of the system.

Song et al. [6] analyzed the effect of torsional and axial vibration coupling at the crankshafts. Large angular displacements occur when the axial and torsional natural frequencies are equal, or, when the first one is two times greater than the second one.

Lacy [7] reported the torsional vibration analysis of four-cylinder gasoline engine. In his model the journals are connected to the main bearings taking into account the elastic properties of the oil film. Boysal and Rahnejat [8] used the same model proposed by Lacy and included the dynamical rigid body influence of all involved inertias on a multi-body model to analyze the noise generated by the vibrations.

The torsional damping coefficients of internal combustion engines were initially estimated by researchers like Den Hartog [9] and Ker Wilson [10]. These parameters were obtained from empirical determinations and in most cases, were not accurate, generating great variations at the dynamic response of the analyzed systems.

Theoretical and hybrid models to estimate the damping coefficients, were proposed by Iwamoto and Wakabayashi [11], which consider analytical relations between the damping and other measurable parameters of the engines.

Wang and Lim [12] obtained accurate estimation of the absolute damping of a single-cylinder engine motored by an electric motor. The first two mode shapes of the system were considered and the damping coefficients were obtained in function of the crank angle.

Y. Honda and T. Saito [13] studied the torsional vibrations at a six-cylinder Diesel engine with a rubber damper to reduce the vibratory effects. The transition state matrix methodology was considered and it was observed that the torsional stiffness of the rubber torsional vibration damper (TVD) is more significant for the system characteristics than the engine internal damping and even TVD damping. This stiffness is determined mainly by the rubber geometry and its chemical properties.

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Maragonis, I.E. [14] realized a research where the variation of the excitation load through the cylinders, due to the wear of piston rings and liner was considered.

II. Introduction

A crankshaft is subjected to many periodical dynamic loads, generating vibrations and consequently stresses that shall be quantified to ensure the structural integrity of the component.

Nowadays, due to technical, commercial and environmental requirements, the internal combustion engines must operate with higher cylinder pressures and the components shall be optimized for the best performance.

Modern calculation methods allow a precise determination of the stress level at the crankshafts critical regions, as well as the evaluation of the fatigue strength. By this way, it is possible to consider safety factors that guarantee sufficient reliability to avoid structural failures and no over-sized components.

Initially, an analysis considering no TVD was performed to adjust and calibrate the engine internal damping and the natural frequencies of the system. In the second step it was considered a rubber TVD, where its power dissipation capability was checked for structural integrity verification. Finally, the calculations were done considering a viscous damper at the system.

Complete torsional vibration analysis (TVA) including the calculation of the vibration amplitudes at the crankshaft front-end, actuating torques at rear and front bolted connections and damper power dissipation will be performed for the mentioned cases.

Crankshaft torsional vibrations occur on internal combustion engines due to the periodical nature of the actuating torque. Basically, the TVA starts obtaining a mathematical model that represents the system dynamical characteristics, such as: Inertias, torsional stiffness and damping. Then, the excitation torque shall be calculated considering the gas load and inertia forces of the moving parts and a Fourier series expansion of this torque shall be performed. The obtained harmonics shall be applied at the corresponding crank throws, considering the ignition timing of the engine.

III. Theoretical modeling

The crankshafts are subjected to torsional, axial and flexural vibrations, due to the periodic nature of the excitation loading. In this paper, only torsional vibrations analysis is performed and to do these verifications, it is necessary to determine an equivalent mathematical model of the system.

One type of analysis is performed considering a viscous TVD assembled to the crankshaft. Another one considers a double mass rubber damper to reduce the torsional amplitudes. The figure 1 shows the model for a single mass viscous damper TVA, while the figure 2 presents the considered model for a double mass rubber damper analysis.

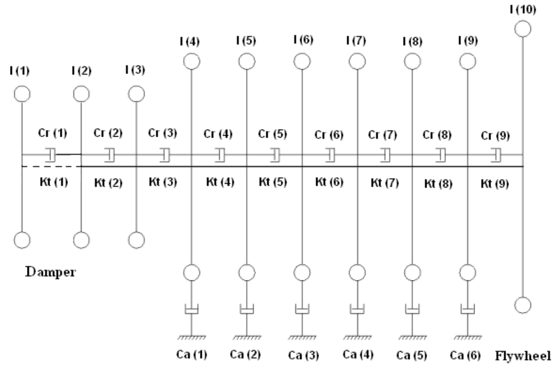

Fig. 1: Equivalent model considering a single mass viscous TVD

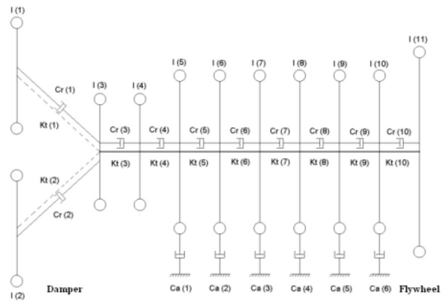

Fig. 2: Equivalent model considering a double mass rubber TVD

A. Inertias

The inertias of the system, such as: flywheel, pulleys, crank throws and TVD rings can be determined by CAD software.

The connecting rod mass shall be divided in two masses. One of them has a purely rotating motion "m_{rb}", while the other one has only oscillating motion "m_{ab}". It is well known that the rotating mass of the connecting rod shall be considered for the crank throw inertia calculation.

The division of the con-rod mass "m_b" including the bolts, bearings and bushing, can be done according to the following methodology:

$$m_{ab} = \frac{m_b \cdot L_2}{L} \quad ; \quad m_{rb} = \frac{m_b \cdot L_1}{L} \qquad (1)$$

Where L is the con-rod length, L_1 and L_2, respectively, the distances from the con-rod mass center to piston pin and crankpin geometric centers.

Usually, the engines have a gear train for power transmission to other devices. The inertia of this complete system shall be considered on the equivalent model. For example, the equivalent inertia of a device motored by gear 2 with a rotational speed n_2 related to gear 1 with a rotational speed n_1 (e.g. crankshaft gear), can be done according to the equation:

$$I_{red} = I_2 \cdot \left(\frac{n_2}{n_1} \right)^2 \quad (2)$$

This reduction shall be done for all motored devices by the gear train referred to the crankshaft gear.

B. Torsional stiffness

The torsional stiffness of all sections of the model can be calculated considering finite element models, where a constant torque is applied at one side of the part and the twist angle is obtained considering that the model is clamped at the other extremity. The relation between the torque and the calculated twist angle is the torsional stiffness that shall be considered at the equivalent model.

B.1 Rubber TVD

The dynamic stiffness of the rubber TVD is determined considering also a finite element model. To perform this calculation we can adopt a dynamic shear modulus of the rubber in the range of: 1.5 MPa ≤ G ≤ 3.0 MPa, according to reference [15]. The Poisson's ratio is: 0.49.

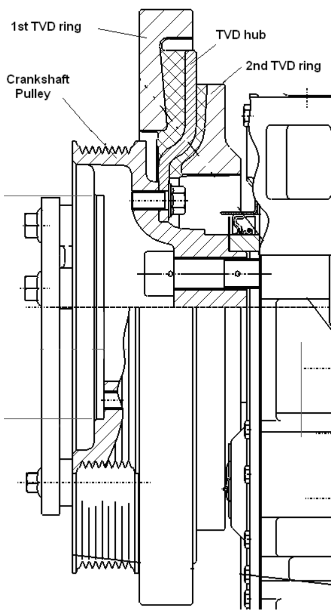

Fig. 3: Double mass rubber damper

B.2 Viscous TVD

The viscous damper torsional stiffness can be determined according to the following methodology, see ref. [16], in function of the silicone kinematic viscosity. The dynamic stiffness is:

$$Kt = Gs \cdot S \quad (3)$$

Where:

$$Gs = G_{01} \cdot e^{k_1} \cdot f^{k_2} \;;\; k_1 = \frac{B_{01}}{T} \;;\; k_2 = \frac{a_{01} - a_{11}}{T} \;;\; f = j \cdot n .$$

S – clearance factor [m³]
T – absolute mean temperature of the silicone film [K]
j – order number
n – engine speed [s^{-1}]
G_{01}, B_{01}, a_{01}, a_{11}: numerical values for these factors can be obtained at reference [16].

C. Damping coefficients

The relative damping coefficients of the system "Cr" can be obtained from the loss angle property, as will be shown. The loss angle calculation can be done by the equation (4), considering that "ω" is the engine angular speed:

$$\chi = \tan \delta = \frac{Cr \cdot \omega}{Kt} \quad (4)$$

At the resonance, we can define the loss factor property:

$$d = \frac{Cr \cdot \omega_n}{Kt} \quad (5)$$

It is possible to observe that at natural frequency "ω_n" the loss factor is equal to the loss angle and with the determined torsional stiffness "Kt", we can determine the relative damping coefficient.

According to the engine type it is possible to know the mean loss factor and the Table I presents the common values of this property. For other engine types see references [16] and [17].

Engine Type	Loss factor (d)
In-line 4 cylinders (TC)	0.055
In-line 6 cylinders (TC)	0.035

TABLE I. 4-stroke Diesel engines loss factors

The absolute damping coefficients, considered at the position of the crank throw inertias, are basically due to the contact between the piston rings and the block and oil films. It is recommended to determine these properties experimentally, running the engine without TVD and measuring the torsional vibration amplitudes at the dynamometer. After, the calculated vibration amplitudes shall be adjusted to the measured ones. In this specific case we determined a value of 2.0 N.m.s/rad for this property.

C.1 Rubber TVD

For the determination of rubber TVD relative damping coefficient, we can adopt a loss factor in the range of: 0.15 ≤ d ≤ 0.25, according to reference [16].

C.2 Viscous TVD

The relative damping coefficient is determined as follows:

$$Cr = \frac{Gv \cdot S}{\omega} \tag{6}$$

Where:

$$Gv = G_{02} \cdot e^{k_3} \cdot f^{k_4} \;;\; k_3 = \frac{B_{02}}{T} \;;\; k_2 = \frac{a_{02} - a_{12}}{T} \;;\; f = j \cdot n$$

Where:

$\omega = 2\pi n$ – engine angular speed [rad/s]

D. Excitation torque

The torque is calculated from the resultant tangential force multiplied by the crank radius. Initially, the kinematics of the crank mechanism will be determined for further dynamic loading computation. The presented methodology can be fully revised on references [18] and [19].

Only the resultant tangential force “Ft” shall be computed for the torsional vibration analysis. The other loads, such as resultant radial force “Fr”, are important for the crankshaft structural analysis, but these calculations are out of the scope of this work. The resultant tangential force is obtained considering the gas load and the inertia forces of the system.

The gas load can be obtained by the equation:

$$Fg = \frac{\pi \cdot dp^2}{4} \cdot p(\alpha) \tag{7}$$

Where:

dp – piston diameter [mm]

p – cylinder pressure [MPa]

The tangential gas load can be computed according to the following equation:

$$F_{tg} = F_g \cdot \frac{sin(\alpha + \beta)}{\cos\beta} \;;\; sin\beta = \lambda \cdot sin\alpha \tag{8}$$

The oscillating inertia force can be obtained as follows:

$$F_{ia} = m_a \cdot r \cdot \omega^2 (\cos\alpha + \lambda \cdot \cos 2\alpha - \frac{\lambda^3}{4}\cos 4\alpha + \frac{9\lambda^5}{128}\cos 6\alpha) \;;$$

$$\lambda = \frac{r}{L} \tag{9}$$

Where:

m_a – oscillating masses

α – crank angle

By the same way, the tangential inertia force is:

$$F_{ta} = F_{ia} \cdot \frac{\sin(\alpha + \beta)}{\cos\beta} \tag{10}$$

Thus, the resultant tangential force is:

$$F_t = F_{tg} + F_{ta} \tag{11}$$

Finally, the excitation torque {T(t)} can be determined just multiplying the resultant tangential force by the crankshaft radius:

$$T(t) = M_t = F_t \cdot r \tag{12}$$

E. Dynamical characteristics of the system

The differential equation of the system, representing the dynamic characteristics for mechanical vibrations can be determined according to the following procedures. More detailed information about this subject can be found at references [20], [21] and [22].

$$[M] \cdot \{\ddot{\theta}(t)\} + [C] \cdot \{\dot{\theta}(t)\} + [Kt] \cdot \{\theta(t)\} = \{T(t)\} \tag{13}$$

Where:

[M] – inertia matrix, which dimension is 10x10 in case of figure 1 and 11x11 in case of figure 2
[C] – viscous damping matrix
[Kt] – stiffness matrix
{θ(t)} – torsional vibration amplitude
{T(t)} – excitation torque defined by equation 12

The number of degrees of freedom (NDF) of the system is equal to the number of inertias.

The oscillating masses shall be replaced by equivalent inertias, which must have the same kinetic energy of the piston motion. An average inertia will be used for the calculations during one crankshaft revolution. The equation hereunder quantifies this inertia, that must be introduced only at the crank throw inertia matrix positions.

$$I_{alt} = m_a \cdot r^2 \cdot \left(\frac{1}{2} + \frac{\lambda^2}{8}\right) \tag{14}$$

As mentioned before, the excitation torque that actuates on the crankshaft varies according to the crank angle, engine speed and the engine load. This vector is presented by the following equation and in the case of the viscous TVD model, it has 11 degrees of freedom:

$$\{T(t)\} = \{0 \quad 0 \quad 0 \quad 0 \quad M_t^1(t) \quad M_t^2(t) \quad M_t^3(t) \quad M_t^4(t) \quad M_t^5(t) \quad M_t^6(t) \quad 0\}^T \tag{15}$$

The torque that actuates in each crank throw is a periodic excitation function and the solution for this kind of system is found through a finite Fourier series that can be revised at reference [23]. On this study, it will be considered 24 terms for the series expansion.

$$M_t^{\,k}(t) = \frac{A_0^{\,k}}{2} + \sum_{n=1}^{24}\left[C_n^k \cdot e^{in\omega t} + \overline{C}_n^k \cdot e^{-in\omega t}\right] \tag{16}$$

Where C_n^k and $\overline{C}_n^k$ are complex coefficient and its conjugate of the Fourier series, and k = 1, 2, ... ,6 (cylinder number).

F. State equation solution

It is possible to express the dynamic behavior of the crankshaft through the first order differential state equation of the system. For detailed explanation see reference [20]:

$$\dot{x}(t) = A \cdot x(t) + b(t) \quad ; \quad \dot{x}(t) = \begin{Bmatrix} \theta(t) \\ \dot{\theta}(t) \end{Bmatrix} \tag{17}$$

Where:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1} \cdot Kt & -M^{-1} \cdot C \end{bmatrix} \quad b(t) = \begin{Bmatrix} 0 \\ M^{-1} \cdot T(t) \end{Bmatrix} \quad b(t) = \begin{Bmatrix} b1(t) \\ b2(t) \end{Bmatrix}$$

For viscous TVD model:

$$b1(t) = \{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}^T$$

$$b2(t) = M^{-1} \cdot T(t) = \left\{0 \quad 0 \quad 0 \quad 0 \quad \frac{M_t^1(t)}{I(5)} \quad \frac{M_t^2(t)}{I(6)} \quad \frac{M_t^3(t)}{I(7)} \quad \frac{M_t^4(t)}{I(8)} \quad \frac{M_t^5(t)}{I(9)} \quad \frac{M_t^6(t)}{I(10)} \quad 0\right\}^T$$

G. System steady state response calculation

The response of a periodic excited vibratory linear system, represented by its state equation, can be obtained via the fundamental matrix, or transition state matrix, and the convolution integral.

Disregarding the transient and the constant Fourier term and solving the harmonic terms summation, the system response can be obtained as follows:

$$x_n(t) = \theta_n(t) = g_n \cdot e^{in\omega \cdot t} + \overline{g}_n \cdot e^{-in\omega \cdot t} \tag{18}$$

Frequency response vector:

$$g_n = F_n \cdot b_n \quad ; \quad \overline{g_n} = \overline{F_n} \cdot \overline{b_n} \tag{19}$$

Frequency matrix:

$$F_n = (in\omega \cdot I - A)^{-1} \quad ; \quad \overline{F_n} = (-in\omega \cdot I - A)^{-1} \tag{20}$$

Therefore, the global vibration amplitude can be computed by the equation:

$$\theta_j = \sum_{n=1}^{24} \Theta_{n_j} \cdot \cos\left(n\omega \cdot t - \phi_{n_j}\right) \tag{21}$$

Where:

$$\Theta_{n_j} = 2 \cdot \sqrt{[\mathrm{Re}(g_{n_j})]^2 + [\mathrm{Im}(g_{n_j})]^2} = 2 \cdot \left|g_{n_j}\right| \quad ;$$

$$\phi_{n_j} = \mathrm{atan}\frac{-\mathrm{Im}(g_{n_j})}{\mathrm{Re}(g_{n_j})}$$

n = 1 ... 24 (Fourier series term)
j = 1 ... NDF

Knowing the torsional vibration amplitude of two consecutive inertias, it is possible to calculate the actuating torque according to the following equation:

$$T_{j-1} = \left|\theta_j - \theta_{j-1}\right| \cdot Kt_{j-1} \quad ; \quad j = 1 \ldots NDF \tag{22}$$

It is important to note that the constant Fourier term must be added to the calculated torsional vibration torque, taking into account the number of cylinders ahead the considered inertia. For example: The constant Fourier term must be summed 6 times to the calculated torque between the flywheel and the sixth cylinder.

For rubber TVD, it is also possible to calculate the actuating shear stress and maximum deformation of the rubber. The maximum shear stress shall not exceed 0.3 ... 0.4 MPa and its calculation can be done, through the relation of the torque between the damper ring and hub and the rubber section modulus under shear:

$$\tau_j = \frac{\left|\theta_j - \theta_3\right| \cdot Kt_j}{Wt_j} \quad ; \quad j = 1, 2 \text{ (for a double TVD)} \tag{23}$$

The maximum deformation of the rubber shall not exceed 15 ... 20% and its calculation can be done by the following equation, considering that for small angles $tg(\Delta\theta) \cong \Delta\theta$:

$$\varepsilon_j = \frac{\tau_{j\max} \cdot Wt_j}{Kt_j} \cdot \frac{R_j}{e_j} \cdot 100\% \quad ; \quad j = 1, 2 \tag{24}$$

Where:
Wt – rubber section modulus under shear
Kt – rubber torsional stiffness
θ – torsional vibration amplitude
R – maximum radius of the rubber at TVD
e – rubber thickness

These permissible parameters are stipulated by TVD manufactures and its reliability is obtained from many tests at dynamometers and vehicles.

IV. Conclusions

A formulation to calculate the torsional vibration of internal combustion engine crankshafts was presented.

It includes also the influence of rubber or viscous dampers and can be used to estimate the cycle life of the crankshaft and absorbers. The validation of the presented methodology can be found in the paper "Experimental Validation of a Methodology for Torsional Vibration Analysis in Internal Combustion Engines" written by the same authors.

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